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INDEX MATRICES AND OLAP-CUBE
PART 5: INDEX MATRIX OPERATIONS OVER OLAP-CUBE

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Abstract. The aim of On-line Analytical Processing (OLAP) is to extract knowledge from data warehouse. It is used to provide users with the ability to perform dynamic data analysis. The index matrices (IMs) concept has been introduced by Atanassov in 1984. The outlined approach for big data analysis using the IMs concept can be applied to both crisp and fuzzy data and can be expanded to retrieve information to other types of multi-dimensional data. The main contribution of the paper is that is proposed to expand the OLAP-cube data analysis by implementing some of the index matrix operations.

AMS Classification: 11C20.

Keywords and phrases. Index Matrices, Multiplication, OLAP cube, On-line analytical processing, Extended index matrices

1. Introduction

Corporate data has grown rapidly during the last years. Business enterprises prosper or fail according to the sophistication and speed of their information systems [19], and their ability to analyze and synthesize information using those systems. The term "OLAP (Online Analytical Processing)" has introduced in 1993 by Codd [19]. OLAP is the dynamic enterprise analysis required to create, manipulate, animate, and synthesize information from exegetical, contemplative, and formulaic data analysis models. This includes the ability to discern new or unanticipated relationships between variables, the ability to identify the parameters necessary to handle large amounts of data, to create an unlimited number of dimensions (consolidation paths), and to specify cross-dimensional conditions and expressions [19]. The most important characteristic of OLAP is its multidimensional view [1].

The OLAP characteristics have been presented in [19] are:

- Dynamic data analysis
  Dynamic data analysis can provide an understanding of the changes occurring within a business enterprise, and may be used to identify candidate

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solutions to specific business challenges as they are uncovered, and to facilitate the development of future strategic and tactical formulae.

- Four Enterprise Data Models
  Both of these types of data analysis employ data models as their principal tool. The data models used in both static and dynamic data analysis fall into four categories: the categorical model; the exegetical model; the contemplative model; and the formulaic model.
  At the starting point of the continuum is the categorical data model. This model is employed in static data analysis to describe what has gone on before by comparing historical values or behaviors which have typically been stored in the enterprise database.
  Moving along the continuum, the exegetical model reflects what has previously occurred to bring about the state which the categorical model reflects. This tool is primarily a static data analysis model with which the user peels back one or more layers of the onion through subsequent simple queries.
  The third model, the contemplative model, indicates what outcomes might result from the introduction of a specific set of parameters or variances across one or more dimensions of the data model.
  The fourth data model, the formulaic model, is the most dynamic and requires the highest degree of user interaction and associated variable data consolidation. Within this dynamic model, the analyst specifies an outcome, and a starting point, animates the model, and gains insight as to what variables of which dimensions must be manipulated and to what extent, if the if the specified outcome is to be attained.
- Common Enterprise Data
  Frequently, especially in the use of exegetical analysis, the analyst must drill down within a particular data consolidation path to discern why a particular situation is as it is. Referring back to the actual transaction data is often necessary. This data must be readily available to the enterprise user-analyst. Contemporary application of this analysis is for predicting fires [20].
- Synergistic Implementation
  Examination of the collection of functions requiring implementation in order to support OLAP suggests the inclusion of the following:

  – access to the data in the Database management system (DBMS) or access method files;

  – definitions of the data and its dimensions required by the user;

  – the variety of ways and contexts in which the user might wish to view, manipulate, and animate the data model;

  – accessibility to these functions via the end-users customary interface.

An architectural framework is required within which the function could be made to appear as if it were part of the user analysts customary spreadsheet product. This framework has become a significant measure of the efficacy of the multidimensional data analysis product itself, and, in fact, represents the first criterion for evaluating OLAP products.

An enterprises efficiency will be in positive correlation to the quality of its OLAP capability. The IT departments within enterprises need to prepare for and to provide rigorous OLAP support for their enterprises.

In the current paper, which is a continuation of the articles [16, 17, 33, 34], the autors propose to expand the OLAP-cube data analysis by implementing some of the index matrix operations. Some of them operate with large aggregates of multidimensional data from two or more OLAP cubes, including in an uncertain environment. Some of them can be made manually in the software environment of Multidimensional expressions language (MDX). MDX provides a syntax for querying and manipulating the multidimensional data [23].

The concept of IMs was introduced in 1984 in [2, 3]. IMs appear as an auxiliary tool for describing different mathematical objects, but they are mainly used to describe the transitions in generalized nets [2, 4, 9, 25, 27, 28, 29, 36], intuitionistic fuzzy relations and graphs with finite sets of vertices [5, 6, 7, 8, 11] as well as some algorithms for decision making [11, 12, 21, 22, 26, 30].

The rest of this paper is structured as follows: Section 2 defines the three-dimensional extended index matrix (3D-EIM) and three-dimensional multilayer extended index matrix (3D-MLEIM). In Section 3, we describe some of IMs operations and propose to expand the OLAP-cube data analysis by their implementing. Section 4 ofers the conclusion and outlines aspects for future research.

2. Short remarks on 3D-Extended index matrix

Let us recall with a definition of a 3D-extended index matrix from [10, 14, 32].

2.1. **Definition of 3D-EIM.** An Intuitionistic Fuzzy Pair (IFP) [11, 13] is an object with the form $\langle a, b\rangle$, where $a, b \in [0, 1]$ and $a + b \leq 1$, that is used as an evaluation of some object or process. Its components (a and b) are interpreted as degrees of membership and non-membership. Let $\mathcal{I}$ be a fixed set of indixes,

$$\mathcal{I}^n = \{\langle i_1, i_2, ..., i_n\rangle | (\forall j : 1 \leq j \leq n)(i_j \in \mathcal{I})\}$$

and

$$\mathcal{I}^* = \bigcup_{1\leq n\leq\infty} \mathcal{I}^n.$$

Let $\mathcal{X}$ be a fixed set of some objects. In the particular cases, they can be either real numbers, or only the numbers 0 or 1, or logical variables, propositions or predicates, IFPs, functions, etc.

A "3D-Extended Index Matrix" (3D-EIM) with index sets K, L and H, which are a subset of $\mathcal{I}^*$. and elements from set $\mathcal{X}$ is called the object:

$$[K,L,H,\{a_{k_i,l_j,h_g}\}] = \left\{ \begin{array}{c|ccccc} h_g & l_1 & \dots & l_j & \dots & l_n \\ \hline k_1 & a_{k_1,l_1,h_g} & \vdots & a_{k_1,l_j,h_g} & \dots & a_{k_1,l_n,h_g} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ k_i & a_{k_i,l_1,h_g} & \dots & a_{k_i,l_j,h_g} & \dots & a_{k_i,l_n,h_g} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ k_m & a_{k_m,l_1,h_g} & \dots & a_{k_m,l_j,h_g} & \dots & a_{k_m,l_n,h_g} \end{array} \quad | \ h_g \in H \right\}$$

where $K = \{k_1, k_2, \dots, k_m\}$, $L = \{l_1, l_2, \dots, l_n\}$, $H = \{h_1, h_2, \dots, h_f\}$, and for $1 \le i \le m,\ 1 \le j \le n,\ 1 \le g \le f : a_{k_i,l_j,h_g} \in \mathcal{X}$.

Following [10, 32], let $3D - EIM_R$ be the set of all 3D-EIMs with elements being real numbers; $3D - EIM_{\{0,1\}}$ be the set of all $(0,1)$-3D-EIMs with elements being 0 or 1; $3D - EIM_P$ be the set of all 3D-EIMs with elements – predicates; $3D - EIM_{IFP}$ be the set of all 3D-EIMs with elements – IFPs and $3D - EIM_{FE}$ – the set of all 3D-EIMs with elements – 1-argument functions $\in F^{\ 1}$.

2.2. **Definition of 3D-multilayer extended index matrix (3D-MLEIM).** Let us present the definition of 3D-MLEIM A [31, 35] with P-levels (layers) of use of dimension K, Q-levels(layers) of use of dimension L and R-levels(layers) of use of dimension H as follows:

$$A = [(K,P),(L,Q),(H,R),\{a_{K^{(p)}_{i,d},L^{(q)}_{j,b},H^{(r)}_{g,c}}\}]$$

$$= \left\{ \begin{array}{c|ccccc} H_g^{(R)} \in H & L_1^{(Q)} & \dots & L_j^{(Q)} & \dots & L_n^{(Q)} \\ \hline K_1^{(P)} & a_{K_1^{(P)},L_1^{(Q)},H_g^{(R)}} & \dots & a_{K_1^{(P)},L_j^{(Q)},H_g^{(R)}} & \dots & a_{K_1^{(P)},L_n^{(Q)},H_g^{(R)}} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ K_i^{(P)} & a_{K_i^{(P)},L_1^{(Q)},H_g^{(R)}} & \dots & a_{K_i^{(P)},L_j^{(Q)},H_g^{(R)}} & \dots & a_{K_i^{(P)},L_n^{(Q)},H_g^{(R)}} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ K_m^{(P)} & a_{K_m^{(P)},L_1^{(Q)},H_g^{(R)}} & \dots & a_{K_m^{(P)},L_j^{(Q)},H_g^{(R)}} & \dots & a_{K_m^{(P)},L_n^{(Q)},H_g^{(R)}} \end{array} \right\},$$

where

$$K = \{K_1^{(P)}, K_2^{(P)}, \dots, K_i^{(P)}, \dots, K_m^{(P)}\},$$
$$K_i^{(P)} = \left\{K_{i,1}^{(P-1)}, K_{i,2}^{(P-1)}, \dots, K_{i,x}^{(P-1)}, \dots, K_{i,I}^{(P-1)}\right\} \text{ for } 1 \le i \le m$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$K_u^{(1)} = \left\{K_{u,1}^{(0)}, K_{u,2}^{(0)}, \dots, K_{u,U}^{(0)}\right\}$$

i.e. $p-$th layer of dimension K of the multilayer matrix, where $(1 \le p \le P)$, is represented by

$$K_{u_*}^{(p)} = \left\{K_{u_*,1}^{(p-1)}, K_{u_*,2}^{(p-1)}, \dots, K_{u_*,U_*}^{(p-1)}\right\} \text{ for } 1 \le p \le P$$
$$L = \{L_1^{(Q)}, L_2^{(Q)}, \dots, L_j^{(Q)}, \dots, L_n^{(Q)}\},$$
$$L_j^{(Q)} = \{L_{j,1}^{(Q-1)}, L_{j,2}^{(Q-1)}, \dots, L_{j,y}^{(Q-1)}, \dots, L_{j,J}^{(Q-1)}\} \text{ for } 1 \le j \le n$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$L_v^{(1)} = \left\{L_{v,1}^{(0)}, L_{v,2}^{(0)}, \dots, l_{v,V}^{(0)}\right\}$$

i.e. $q-$th layer of dimension Q of the multilayer matrix is represented by

$$L_{v_*}^{(q)} = \left\{L_{v_*,1}^{(q-1)}, L_{v_*,2}^{(q-1)}, \ldots, L_{v_*,V_*}^{(q-1)}\right\} \text{ for } 1 \leq q \leq Q$$

$$H = \{H_1^{(R)}, H_2^{(R)}, \ldots, H_g^{(R)}, \ldots, H_f^{(R)}\},$$

$$H_g^{(R)} = \{H_{g,1}^{(R-1)}, H_{g,2}^{(R-1)}, \ldots, H_{g,z}^{(R-1)}, \ldots, H_{g,G}^{(R-1)}\} \text{ for } 1 \leq g \leq f$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$H_w^{(1)} = \left\{H_{w,1}^{(0)}, H_{w,2}^{(0)}, \ldots, H_{w,W}^{(0)}\right\}$$

i.e. $r-$th layer of dimension H of the multilayer matrix is represented by

$$H_{w_*}^{(r)} = \left\{H_{w_*,1}^{(r-1)}, H_{w_*,2}^{(r-1)}, \ldots, H_{w_*,W_*}^{(r-1)}\right\} \text{ for } 1 \leq r \leq R$$

and $(K, L, H \subset \mathcal{I}^*)$, and for $1 \leq i \leq I,\ 1 \leq j \leq J,\ 1 \leq g \leq G,\ 1 \leq p \leq P,\ 1 \leq q \leq Q,\ 1 \leq r \leq R,\ 1 \leq d \leq I,\ 1 \leq b \leq J,\ 1 \leq c \leq G : a_{K_{i,d}^{(p)}, L_{j,b}^{(q)}, H_{g,c}^{(r)}} \in \mathcal{X},\ K_{i,0}^{(p)} \notin K,$ $L_{j,0}^{(q)} \notin L$ and $H_{g,0}^{(r)} \notin H$.

3. An implementiation of the IMs operations to the data of OLAP-cube

3.1. An implementation of IMs multiplication and transposition to the OLAP-cubes data.

3.1.1. *IMs transposition and multiplication.* Let us be given 3D-EIMs $A = [K, L, H, \{a_{k_i,l_j,h_g}\}]$ and $B[P, Q, R, \{b_{p_r,q_s,e_d}\}]$. There are 6-1 (=3!-1) transposed 3D-EIMs of A [10]:

$[1,3,2]$**-transposition**

$$[K, L, H, \{a_{k_i,l_j,h_g}\}]^{[1,3,2]} = [K, H, L, \{a_{k_i,h_g,l_j}\}];$$

$[2,1,3]$**-transposition**

$$[K, L, H, \{a_{k_i,l_j,h_g}\}]^{[2,1,3]} = [L, K, H, \{a_{l_j,k_i,h_g}\}];$$

$[2,3,1]$**-transposition**

$$[K, L, H, \{a_{k_i,l_j,h_g}\}]^{[2,3,1]} = [L, H, K, \{a_{l_j,h_g,k_i}\}];$$

$[3,1,2]$**-transposition**

$$[K, L, H, \{a_{k_i,l_j,h_g}\}]^{[3,1,2]} = [H, K, L, \{a_{h_g,k_i,l_j}\}];$$

$[3,2,1]$**-transposition**

$$[K, L, H, \{a_{k_i,l_j,h_g}\}]^{[3,2,1]} = [H, L, K, \{a_{h_g,l_j,k_i}\}].$$

There are six different operations “multiplication” between two IMs. The first (standard) operation multiplication, expanded according to the type of the element of the matrix is:

$$A \odot_{(\circ,*)} B = A \odot_{(\circ,*)}^{[1,2,3]} B = [K \cup Q, L \cup P, H \cup R, \{c_{t_u,v_w,x_y}\}], \text{ where}$$

$$c_{t_u,v_w,x_y}$$

$$= \begin{cases} a_{k_i,l_j,h_g}, & \text{for } t_u = k_i \in K \\ & \& \ v_w = l_j \in L - P - Q \\ & \ x_y = h_g \in H \\ & \text{or} \\ t_u = k_i \in K - P - Q & \\ & \& \ v_w = l_j \in L \\ & \& \ x_y = h_g \in H \\ & \\ b_{p_r,q_s,e_d}, & \text{if } t_u = p_r \in P \\ & \& \ v_w = q_s \in Q - L - K \\ & x_y = e_d \in R \\ & \text{or } t_u = p_r \in P \\ & \& \ v_w = q_s \in Q - L - K \\ & \& \ x_y = e_d \in R \\ & \\ \underset{l_j = p_r \in L \cap P}{\circ} a_{k_i,l_j,h_g} * b_{p_r,q_s,e_d}, & \ \text{if } t_u = k_i \in K \\ & \& \ v_w = q_s \in Q \ \& \\ & \ x_y = h_g = e_d \in H \cap R \\ & \\ 0 \qquad \text{for } A, B \in 3D - EIM_{\mathcal{R}}, & \text{elsewhere} \\ \qquad\ \ \text{or } A, B \in 3D - EIM_{\{0,1\}} & \\ F \qquad \text{for } A, B \in 3D - EIM_{\mathcal{P}} & \\ \langle 0, 1 \rangle \ \text{for } A, B \in 3D - EIM_{\mathcal{IFP}} & \end{cases}$$

and

$$\langle \circ, * \rangle \in \begin{cases} \{\langle +, \times \rangle, \langle \max, \min \rangle, \langle \min, \max \rangle\}, & \text{if } A \in 3D - EIM_{\mathcal{R}} \\ \{\langle \max, \min \rangle, \langle \min, \max \rangle\}, & \text{if } A \in 3D - EIM_{\{0,1\}} \\ \{\langle \wedge, \vee \rangle, \langle \wedge, \vee \rangle\}, & \text{if } A \in 3D - EIM_{\mathcal{P}} \\ & \text{or } A \in 3D - EIM_{\mathcal{IFP}} \end{cases}.$$

3.1.2. *An implementation of IMs multiplication and transposition between n OLAP-cubes data.* For this purposes we use two OLAP cubes “Bookshops” and “Bookshops1” with similar structure, which is given in [33]. They contain information for the book sales in different bookshops (managed by different regional managers) in different locations. The structure of the two cubes “Bookshops” is visualized on Fig. 1.

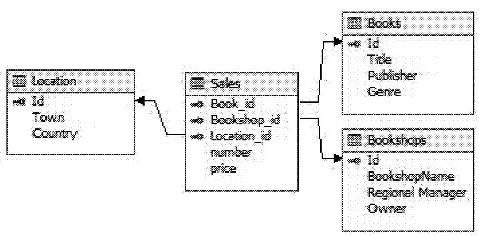

Fig. 1. Star schema “Bookshops”

The fact table "Sales" and the dimensional tables BooksId, Title, Publisher, Genre, Price, BookshopsId, Bookshop Name, Regional Manager, Owner and LocationId, Town, Country are constructed. The measures are "NumberSales" and "Sales Count". The hierarchical structures of the dimensions are presented in [33].

In terms of 3D-EIMs the above defined two OLAP-cubes in this section are converted into the following two IMs $A = [K, L, H, \{a_{k_i,l_j,h_g}\}]$ and $B[P, Q, R, \{b_{p_r,q_s,e_d}\}]$. After applying the IMs operation "multiplication" between two IMs, the following IM (OLAP-cube) shown on Fig. 2 is obtained.

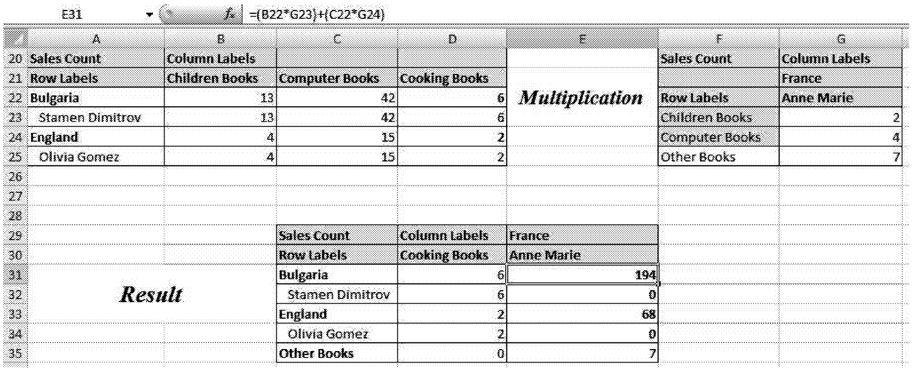

FIG. 2. Index matrix operation "Multiplication" over OLAP cubes

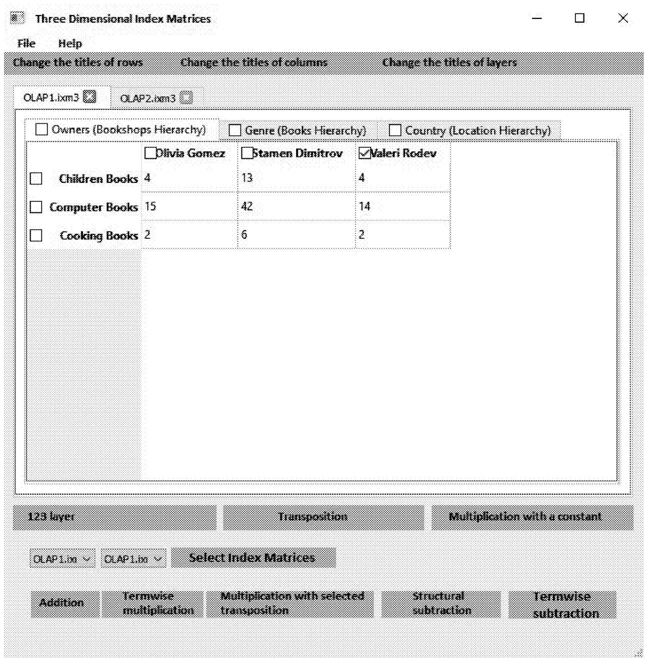

FIG. 3. The input matrix (OLAP-cube) A

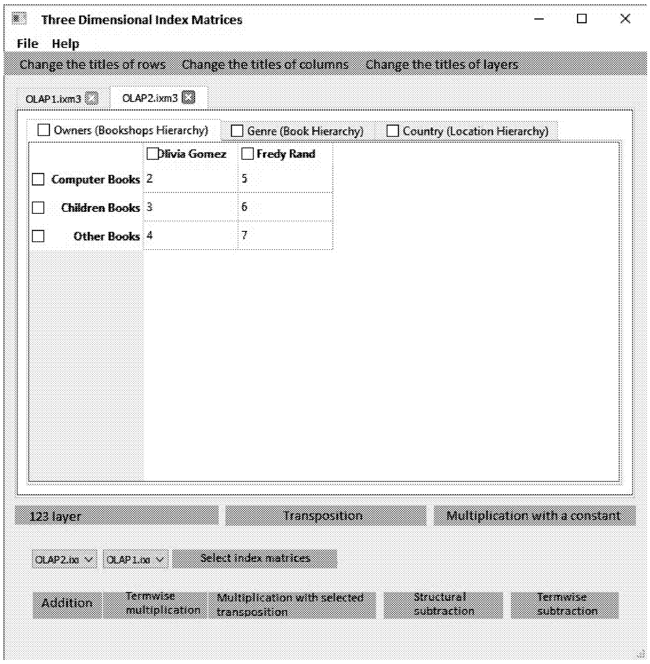

FIG. 4. The input matrix (OLAP-cube) B

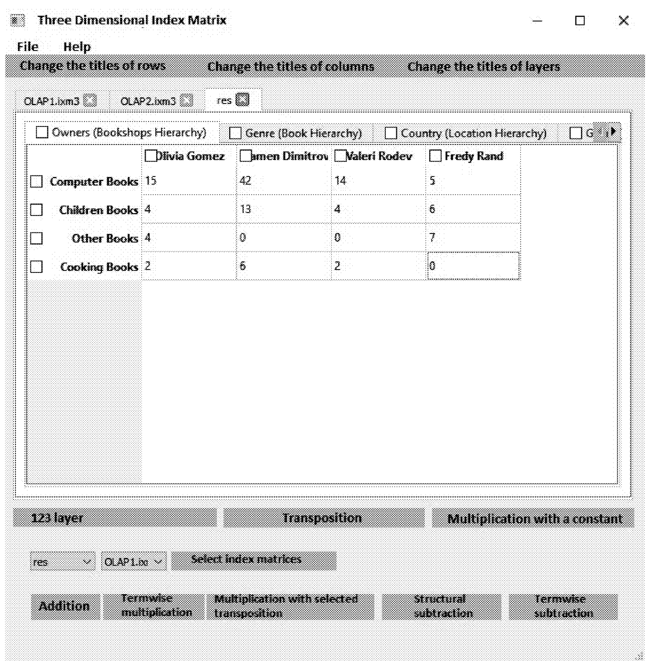

FIG. 5. The resultant matrix (OLAP-cube) C

The operation is performed in Excel environment using OLAP extension. The connection to the SQL Server Analysis Services (SSAS) Bookshops cube is made. The operation "multiplication" between two OLAP-cubes, presented by two IMs A and B, is executed by D. Mavrov software for IMs. The first input IM A is shown on Fig. 3. The second input IM B is shown on Fig. 4. The resultant IM (OLAP-cube) C is given on Fig. 5.

3.2. **An implementing of IM automatic reduction and decomposition to the OLAP-cube data.**

3.2.1. *IM automatic reduction.*

- Automatic reduction

Let us be given 3D-EIM $A = [K, L, H, \{a_{k_i,l_j,h_g}\}]$. The definition of the operation for an EIM A is

$$@(A) = [P, Q, \{b_{p_r,q_s,d_e}\}],$$

where $P \subseteq K, Q \subseteq L, R \subseteq H$ are index sets with the following properties: $(\forall k \in K - P)(\forall l \in L)(\forall h \in H)(a_{k_i,l_j,h_g} = \perp)$
$\&\ (\forall k \in K)(\forall l \in L - Q)(\forall h \in H)(a_{k_i,l_j,h_h} = \perp)$
$\&\ (\forall k \in K)(\forall l \in L)(\forall h \in H - K)(a_{k_i,l_j,h_g} = \perp)$
$\&\ (\forall k \in K)(\forall l \in L)(\forall h \in H - L)(a_{k_i,l_j,h_g} = \perp)$
$\&\ (\forall k \in K - H)(\forall l \in L)(\forall h \in H)(a_{k_i,l_j,h_g} = \perp)$
$\&\ (\forall k \in K)(\forall l \in L - H)(\forall h \in H)(a_{k_i,l_j,h_g} = \perp)$
$\&\ (\forall p_r = a_i \in P)(\forall q_s = b_j \in Q)(\forall d_e = h_g \in H)$
$(b_{p_r,q_s,d_e} = a_{k_i,l_j,h_g})$.

In [34] has been defined "**automatic zero-reduction**"

$$@_0(A) = [P, Q, \{b_{p_r,q_s,d_e}\}]$$

where $P \subseteq K, Q \subseteq L, R \subseteq H$ are index sets with following properties:

$$(\forall k \in K - P)(\forall l \in L)(\forall h \in H)(a_{k_i,l_j,h_g} = 0)$$

$$\&\ (\forall k \in K)(\forall l \in L - Q)(\forall h \in H)(a_{k_i,l_j,h_h} = 0)$$

$$\&\ (\forall k \in K)(\forall l \in L)(\forall h \in H - K)(a_{k_i,l_j,h_g} = 0)$$

$$\&\ (\forall k \in K)(\forall l \in L)(\forall h \in H - L)(a_{k_i,l_j,h_g} = 0)$$

$$\&\ (\forall k \in K - H)(\forall l \in L)(\forall h \in H)(a_{k_i,l_j,h_g} = 0)$$

$$\&\ (\forall k \in K)(\forall l \in L - H)(\forall h \in H)(a_{k_i,l_j,h_g} = 0)$$

$$\&\ (\forall p_r = a_i \in P)(\forall q_s = b_j \in Q)(\forall d_e = h_g \in H)(b_{p_r,q_s,d_e} = a_{k_i,l_j,h_g}).$$

The operation automatic reduction automatically deletes the attributes and rows containing only the null values. The removed attribute, member will not to be available in the selected level of the hierarchy. Usually this step is made using the visual tools of the software. After the attribute removing the dimension or whole cube have to be processed.

3.2.2. *An application of IM automatic reduction to the OLAP-cube data.* Let us apply the OLAP-cube operation "slice" [16, 18] to data, saved in OLAP-cube A. The result is shown on Fig. 6.

	Roses 1	Helicon 3	Helikon 1	Helikon 2	Penguins 1	Penguins 2
Circles	2	2	4	3	1	(null)
Mulan	1	1	2	1	2	(null)
Bears Dont Read	1	2	4	3	2	(null)
Sparky	2	3	3	2	1	(null)
Microsoft PowerPoint 2013 - Step by Step	2	1	2	2	2	(null)
Python-practical programming	1	3	1	2	1	(null)
R in Action	3	2	1	2	3	(null)
The C# Programming Yellow Book	2	10	3	2	1	(null)
Business Analysis	2	4	1	2	3	(null)
Introducing Microsoft SQL server 2014	2	4	1	1	1	(null)
Programming Microsoft SQL Server 2012	4	9	1	2	2	(null)
C# 5.0 in a Nutshell: The Definitive Reference	2	7	2	1	1	(null)
Discovering Statistics Using R	7	6	2	4	1	(null)
Hadoop: The Definitive Guide	3	4	1	1	1	(null)
JavaScript: The Definitive Guide	3	5	2	1	2	(null)
JavaScript: The Good Parts	1	4	2	2	1	(null)
Learning Redis	1	13	2	2	2	(null)
Introduction in Computer Design	4	3	2	3	4	(null)
La Repertoire de la Cuisine	1	1	1	5	4	(null)
My Paris Kitchen: Recipes and Stories	2	1	6	1	3	(null)

FIG. 6. The OLAP-cube slice

	Roses 1	Helicon 3	Helikon 1	Helikon 2	Penguins 1
Circles	2	2	4	3	1
Mulan	1	1	2	1	2
Bears Dont Read	1	2	4	3	2
Sparky	2	3	3	2	1
Microsoft PowerPoint 2013 - Step by Step	2	1	2	2	2
Python-practical programming	1	3	1	2	1
R in Action	3	2	1	2	3
The C# Programming Yellow Book	2	10	3	2	1
Business Analysis	2	4	1	2	3
Introducing Microsoft SQL server 2014	2	4	1	1	1
Programming Microsoft SQL Server 2012	4	9	1	2	2
C# 5.0 in a Nutshell: The Definitive Reference	2	7	2	1	1
Discovering Statistics Using R	7	6	2	4	1
Hadoop: The Definitive Guide	3	4	1	1	1
JavaScript: The Definitive Guide	3	5	2	1	2
JavaScript: The Good Parts	1	4	2	2	1
Learning Redis	1	13	2	2	2
Introduction in Computer Design	4	3	2	3	4
La Repertoire de la Cuisine	1	1	1	5	4
My Paris Kitchen: Recipes and Stories	2	1	6	1	3

FIG. 7. The IM automatic zero-reduction

We see that one of the columns of the result subcube contains only null values. There are no operations that can be applied to the OLAP cube to reduce this column. We propose to use the index-matrix automatic null-reduction operation.

Fig. 7 presents the resultant IM (OLAP-cube) after applying the automatic zero-reduction operation. The action of this operation differs from that of the operation “projection” (see [16]), which we have used to present the slice and dice operations on the OLAP-cube “Bookshops”. The operation “reduction” deletes attributes.

3.3. **An application of IM substitution to the OLAP-cube data.**

3.3.1. *IM substitution.* Let 3D-EIM $A = [K, L, \{a_{k,l}\}]$ be given. Local substitution over the IM is defined for the couples of indices (p, k) and/or (q, l), and/or (r, h), respectively, by [10, 35]

$$\left[\frac{p}{k}; \perp; \perp\right] A = [(K - \{k\}) \cup \{p\}, L, H, \{a_{k,l,h}\}],$$

$$\left[\perp; \frac{q}{l}; \perp\right] A = [K, (L - \{l\}) \cup \{q\}, H, \{a_{k,l,h}\}],$$

$$\left[\perp; \perp; \frac{r}{h}\right] A = [K, L, (H - \{h\}) \cup \{r\}, H, \{a_{k,l,h}\}].$$

Let the sets of indices $P = \{p_1, p_2, ..., p_m\}$, $Q = \{q_1, q_2, ..., q_n\}$, $R = \{r_1, r_2, ..., r_s\}$ be given, where $m = card(K), n = card(L), s = card(H)$.

For them we define sequentially:

$$\left[\frac{P}{K}; \perp; \perp\right] A = \left[\frac{p_1}{k_1}\frac{p_2}{k_2}...\frac{p_n}{k_n}; \perp; \perp\right] A,$$

$$\left[\perp; \frac{Q}{L}; \perp\right] A = \left[\perp; \frac{q_1}{l_1}\frac{q_2}{l_2}...\frac{q_n}{l_n}; \perp\right] A,$$

$$\left[\perp; \perp; \frac{R}{H}\right] A = \left[\perp; \frac{r_1}{h_1}\frac{r_2}{h_2}...\frac{r_s}{h_s}; \perp\right] A,$$

$$\left[\frac{P}{K}; \frac{Q}{L}; \frac{R}{H}\right] A = \left[\frac{p_1}{k_1}\frac{p_2}{k_2}...\frac{p_m}{k_m}; \frac{q_1}{l_1}\frac{q_2}{l_2}...\frac{q_n}{l_n}; \frac{r_1}{h_1}\frac{r_2}{h_2}...\frac{r_n}{h_s}\right] A = [P, Q, R, \{a_{k,l,h}\}].$$

3.3.2. *An application of IM substitution to the OLAP-cube data.* When we modify or delete a dimension in SQL Server Data Tools (SSDT) the changes are not available to users until the dimension will be processed. In the first example on the Fig. 8, the dimension “Location” is changed with the dimension “Time”. The operation is made in “Designer tool”. The cube is processed.

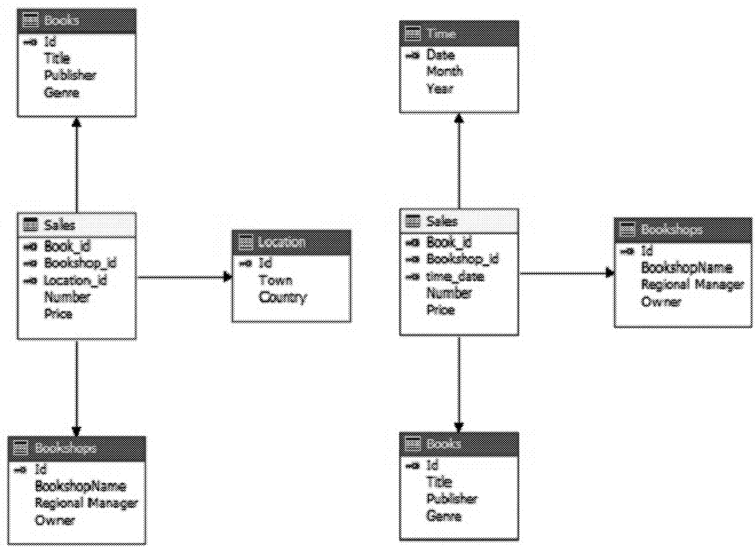

FIG. 8. The IM substitution to OLAP-cube

In the second example on the Fig. 9, the member "Month" of the dimension "Time" is changed with the member "Quarter". The cube is processed.

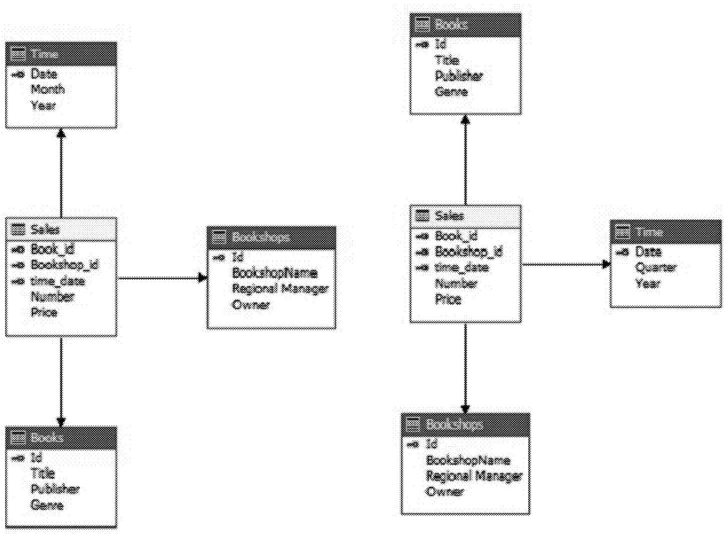

FIG. 9. The IM member substitution to OLAP-cube

Fig. 10 presents the updated structure of the OLAP-cube "Bookshops" after the application of the operation IM "substitution". The cube is processed.

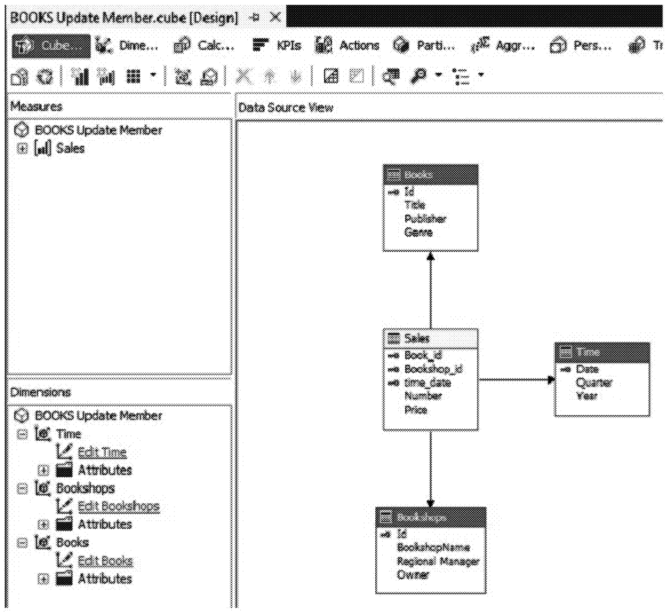

FIG. 10. The updated structure of the OLAP-cube "Bookshops"

3.4. **An application of IM composition and decomposition to the OLAP-cube data.**

3.4.1. *IM composition and decomposition.* Definition 1: In the case of 3D-EIM A:

- Composition

Let the index set $\mathcal{I}^*$ and the set $\mathcal{X}$ be fixed and let 3D-EIMs $A_1, A_2, \ldots, A_n$ over both sets be given. Let for s $(1 \le s \le n)$:

$$A_s = [K^s, L^s, H^s, \{a_{s,k_{s,i},l_{s,j},h_{s,g}}\}]$$

$$= \begin{array}{c|ccccc} h_{s,g} \in H^s & l_{s,1} & \ldots & l_{s,j} & \ldots & l_{s,n_s} \\ \hline k_{s,1} & a_{s,k_{s,1},l_{s,1},h_{s,g}} & \ldots & a_{s,k_{s,1},l_{s,j},h_{s,g}} & \ldots & a_{s,k_{s,1},l_{s,n_s},h_{s,g}} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ k_{s,i} & a_{s,k_{s,i},l_{s,1},h_{s,g}} & \ldots & a_{s,k_{s,i},l_{s,j},h_{s,g}} & \ldots & a_{s,k_{s,i},l_{s,n_s},h_{s,g}} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ k_{s,m} & a_{s,k_{s,m},l_{s,1},h_{s,g}} & \ldots & a_{s,k_{s,m},l_{s,j},h_{s,g}} & \ldots & a_{s,k_{s,m},l_{s,n_s},h_{s,g}} \end{array}.$$

Following [35] the definition of the operation "**composition**" is:

$$\flat\{A_s | 1 \le s \le n\} = [\ \bigcup_{s=1}^{n} K^s,\ \bigcup_{s=1}^{n} L^s,\ \bigcup_{s=1}^{n} H^s,$$

$$\{\langle c_{1,t_{1,u},v_{1,w},d_{1,e}}, c_{2,t_{2,u},v_{2,w},d_{2,e}}, \ldots, c_{n,t_{n,u},v_{n,w},d_{n,e}} \rangle\}],$$

where for r $(1 \le r \le n)$:

$$c_{r,t_{r,u},v_{r,w},d_{r,e}} = \begin{cases} a_{r,k_{r,i},l_{r,j},h_{r,g}}, & \text{if } t_{r,u} = k_{r,i} \in K^r, v_{r,w} = l_{r,j} \in L^r \\ & \text{and } d_{r,e} = h_{r,g} \in H^r \\ \perp, & \text{otherwise} \end{cases},$$

where symbol $\perp$ denotes the lack of some components in the definition or empty matrix elements. We give an example from [10]. Let the EIMs A_1, A_2 have the forms

$$A_1 = \begin{array}{c|cccc} & d & e & f & g \\ \hline a & 1 & 2 & \perp & 3 \\ b & 4 & \perp & 5 & \perp \\ c & 6 & 7 & \perp & 8 \end{array}, \qquad A_2 = \begin{array}{c|ccc} & d & i & f \\ \hline a & 11 & \perp & 12 \\ c & \perp & 13 & 14 \\ h & 15 & \perp & \perp \end{array},$$

respectively.

Then

$$A = \flat\{A_1, A_2\} = \begin{array}{c|ccccc} & d & e & f & g & i \\ \hline a & \langle 1, 11\rangle & \langle 2, \perp\rangle & \langle \perp, 12\rangle & \langle 3, \perp\rangle & \langle \perp, \perp\rangle \\ b & \langle 4, \perp\rangle & \langle \perp, \perp\rangle & \langle 5, \perp\rangle & \langle \perp, \perp\rangle & \langle \perp, \perp\rangle \\ c & \langle 6, \perp\rangle & \langle 7, \perp\rangle & \langle \perp, 14\rangle & \langle 8, \perp\rangle & \langle \perp, 13\rangle \\ h & \langle \perp, 15\rangle & \langle \perp, \perp\rangle & \langle \perp, \perp\rangle & \langle \perp, \perp\rangle & \langle \perp, \perp\rangle \end{array}.$$

- Decomposition

Let $\mathcal{X}$ be a set of n-dimensional vectors and A be an 3D-EIM with elements from set $\mathcal{X}$. Then

$$Pr_s(A) = \begin{cases} I_\emptyset, & \text{if } s \le 0 \text{ or } s > n \\ A_s, & \text{otherwise} \end{cases},$$

where $A_s = [K^s, L^s, H^s, \{a^s_{k_i,l_j,h_g}\}]$ and $a^s_{k_i,l_j,h_g}$ is the s-th component of vector $\langle a^1_{k_i,l_j,h_g}, a^2_{k_i,l_j,h_g}, ..., a^n_{k_i,l_j,h_g}\rangle$ that is an element of A.

The definition of the operation "decomposition" ($\sharp$), which is opposite of "composition" ($\flat$) is:

$$\sharp(A) = \{@Pr_s(A) | 1 \le s \le n\}.$$

Definition 2: In the case of $3D$-MLEIM $A = [K, L, H, \{a_{K^{(p)}_{i,d}, L^{(q)}_{j,b}, H^{(r)}_{g,c}}\}]$:

- Composition

Let index set $\mathcal{I}^*$ and the set $\mathcal{X}$ be fixed and let 3D-MLEIMs $A_1, A_2, \dots, A_n$ over both sets be given. Let for s $(1 \le s \le n)$:

$$A_s = [(K_s, P_s), (L_s, Q_s), (H_s, R_s), \{a_{s,K^{(p_s)}_{s,i,d}, L^{(q_s)}_{s,j,b}, H^{(r_s)}_{s,g,c}}\}] =$$

$$\left\{ \begin{array}{c|ccccc} H^{(R_s)}_{s,g} \in H_s & L^{(Q_s)}_{s,1} & \dots & L^{(Q_s)}_{s,j} & \dots & L^{(Q_s)}_{s,n} \\ \hline K^{(P_s)}_{s,1} & a_{s,K^{(P_s)}_{s,1}, L^{(Q_s)}_{s,1}, H^{(R_s)}_{s,g}} & \dots & a_{s,K^{(P_s)}_{s,1}, L^{(Q_s)}_{s,j}, H^{(R_s)}_{s,g}} & \dots & a_{s,K^{(P_s)}_{s,1}, L^{(Q_s)}_{s,n}, H^{(R_s)}_{s,g}} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ K^{(P_s)}_{s,i} & a_{s,K^{(P_s)}_{s,i}, L^{(Q_s)}_{s,1}, H^{(R_s)}_{s,g}} & \dots & a_{s,K^{(P_s)}_{s,i}, L^{(Q_s)}_{s,j}, H^{(R_s)}_{s,g}} & \dots & a_{s,K^{(P_s)}_{s,i}, L^{(Q_s)}_{s,n}, H^{(R_s)}_{s,g}} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ K^{(P_s)}_{s,m} & a_{s,K^{(P_s)}_{s,m}, L^{(Q_s)}_{s,1}, H^{(R_s)}_{s,g}} & \dots & a_{s,K^{(P_s)}_{s,m}, L^{(Q_s)}_{s,j}, H^{(R_s)}_{s,g}} & \dots & a_{s,K^{(P_s)}_{s,m}, L^{(Q_s)}_{s,n}, H^{(R_s)}_{s,g}} \end{array} \right\},$$

where

$$K_s = \{K_{s,1}^{(P_s)}, K_{s,2}^{(P_s)}, \dots, K_{s,i}^{(P_s)}, \dots, K_{s,m}^{(P_s)}\},$$

$$K_{s,i}^{(P_s)} = \left\{K_{s,i,1}^{(P_s-1)}, K_{s,i,2}^{(P_s-1)}, \dots, K_{s,i,x}^{(P_s-1)}, \dots, K_{s,i,I}^{(P_s-1)}\right\} \text{ for } 1 \le i \le m$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$K_{s,u}^{(1)} = \left\{K_{s,u,1}^{(0)}, K_{s,u,2}^{(0)}, \dots, K_{s,u,U}^{(0)}\right\}$$

i.e. p_s-th layer of dimension K_s of the multilayer matrix, where $(1 \le p_s \le P_s)$, is represented by

$$K_{s,u_*}^{(p_s)} = \left\{K_{s,u_*,1}^{(p_s-1)}, K_{s,u_*,2}^{(p_s-1)}, \dots, K_{s,u_*,U_*}^{(p_s-1)}\right\} \text{ for } 1 \le p_s \le P_s$$

$$L_s = \{L_{s,1}^{(Q_s)}, L_{s,2}^{(Q_s)}, \dots, L_{s,j}^{(Q_s)}, \dots, L_{s,n}^{(Q_s)}\},$$

$$L_{s,j}^{(Q_s)} = \{L_{s,j,1}^{(Q_s-1)}, L_{s,j,2}^{(Q_s-1)}, \dots, L_{s,j,y}^{(Q_s-1)}, \dots, L_{s,j,J}^{(Q_s-1)}\} \text{ for } 1 \le j \le n$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$L_{s,v}^{(1)} = \left\{L_{s,v,1}^{(0)}, L_{s,v,2}^{(0)}, \dots, l_{s,v,V}^{(0)}\right\}$$

i.e. q_s-th layer of dimension Q_s of the multilayer matrix is represented by

$$L_{s,v_*}^{(q_s)} = \left\{L_{s,v_*,1}^{(q_s-1)}, L_{s,v_*,2}^{(q_s-1)}, \dots, L_{s,v_*,V_*}^{(q_s-1)}\right\} \text{ for } 1 \le q_s \le Q_s$$

$$H_s = \{H_{s,1}^{(R_s)}, H_{s,2}^{(R_s)}, \dots, H_{s,g}^{(R_s)}, \dots, H_{s,f}^{(R_s)}\},$$

$$H_{s,g}^{(R_s)} = \{H_{s,g,1}^{(R_s-1)}, H_{g,2}^{(R_s-1)}, \dots, H_{s,g,z}^{(R_s-1)}, \dots, H_{s,g,G}^{(R_s-1)}\} \text{ for } 1 \le g \le f$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$H_{s,w}^{(1)} = \left\{H_{s,w,1}^{(0)}, H_{s,w,2}^{(0)}, \dots, H_{s,w,W}^{(0)}\right\}$$

i.e. r_s-th layer of dimension H of the multilayer matrix is represented by

$$H_{s,w_*}^{(r_s)} = \left\{H_{s,w_*,1}^{(r_s-1)}, H_{s,w_*,2}^{(r_s-1)}, \dots, H_{s,w_*,W_*}^{(r_s-1)}\right\} \text{ for } 1 \le r_s \le R_s$$

and $(K_s, L_s, H_s \subset \mathcal{I}^*)$, and for $1 \le i \le I,\ 1 \le j \le J,\ 1 \le g \le G,\ 1 \le p_s \le P_s,\ 1 \le q_s \le Q_s,\ 1 \le r_s \le R_s,\ 1 \le s \le n,\ 1 \le d \le I,\ 1 \le b \le J,\ 1 \le c \le G:$ $a_{s,K_{s,i,d}^{(p_s)},L_{s,j,b}^{(q_s)},H_{s,g,c}^{(r_s)}} \in \mathcal{X}$.

The definition of the operation "**composition**" is as follows:

$$\flat\{A_s|1 \le s \le n\} = [(\bigcup_{s=1}^{n} K_s, \bigcup_{s=1}^{n} P_s), (\bigcup_{s=1}^{n} L_s, \bigcup_{s=1}^{n} Q_s), (\bigcup_{s=1}^{n} H_s, \bigcup_{s=1}^{n} R_s),$$

$$\{\langle \phi_{1,K_{1,i*,d*}^{(p*1)},L_{1,j*,b*}^{(q*1)},H_{1,g*,c*}^{(r*1)}}, \phi_{2,K_{2,i*,d*}^{(p*2)},L_{2,j*,b*}^{(q*2)},H_{2,g*,c*}^{(r*2)}}, \dots, \phi_{n,K_{n,i*,d*}^{(p*n)},L_{n,j*,b*}^{(q*n)},H_{n,g*,c*}^{(r*n)}} \rangle\}],$$

where for s $(1 \le s \le n)$:

$$\phi_{s,K_{s,i*,d*}^{(p*_s)},L_{s,j*,b*}^{(q*_s)},L_{s,j*,b*}^{(q*_s)}} = \begin{cases} a_{s,K_{s,i,d}^{(p_s)},L_{s,j,b}^{(q_s)},H_{s,g,c}^{(r_s)}}, & \text{if } K_{s,i*,d*}^{(p*_s)} = K_{s,i,d}^{(p_s)} \in K_s^{p_s}, \\ & L_{s,j*,b*}^{(q*_s)} = L_{s,j,b}^{(q_s)} \in L_s^{q_s}, \\ & \text{and } H_{s,g*,c*}^{(r*_s)} = H_{s,g,c}^{(r_s)} \in H_s^{r_s} \\ \perp, & \text{otherwise} \end{cases}.$$

3.4.2. *An implementation of IM decomposition (composition) to the OLAP-cube data.* The "Drill-across" combines the measures of two union comparable OLAP cubes with the same dimensions and levels, i.e. cubes with equal number of dimensions of the same domain and different measures. "Drill-across" between two cubes where they have the same or equal dimension attributes is natural join between corresponding dimension attributes [24]. If the dimension attributes are conformed but they have intersecting values, then the data may be lost during the join. In [17] has been presented the operation "Drill across" in terms of IMs, using the operation "composition" of IMs. The opposite operation of "drill-across" to the OLAP-cube "Bookshops", is presented by the operation "decomposition" of IMs. The operation "decomposition" split the values from the drill-across operation. The result contains the initial cubes.

The following MDX query 1 presents the drill-across operation using measures "Number" and "Sales Count" and the dimensions "Books", "Bookshops" and "Location".

- *MDX query 1:*

```
SELECT NON EMPTY
CrossJoin(Hierarchize(DrilldownMember
({{{DrilldownLevel([Books].[Hierarchy].[All]})}}},
{[Books].[Hierarchy].[Publisher].&[HarperCollins]})),
{[Measures].[Number],[Measures].[Sales Count]})
DIMENSION PROPERTIES PARENT_UNIQUE_NAME,
[Books].[Hierarchy].[Publisher].[Genre],
[Books].[Hierarchy].[Title].[Publisher] ON COLUMNS,
NON EMPTY CrossJoin(Hierarchize(DrilldownMember(
{{{DrilldownLevel([Bookshops].[Hierarchy].[All]})}}},
{[Bookshops].[Hierarchy].[Regional Manager].&[Richard Gray]})),
Hierarchize({DrilldownLevel({[Location].[Hierarchy].[All]})}))
DIMENSION PROPERTIES PARENT_UNIQUE_NAME,
[Bookshops].[Hierarchy].[Regional Manager].[Owner],
[Bookshops].[Hierarchy].[Bookshop Name].[Regional Manager] ON ROWS
FROM [Bookshops3] CELL PROPERTIES VALUE,
FORMAT_STRING, LANGUAGE, BACK_COLOR, FORE_COLOR, FONT_FLAGS
```

Result: The result of the MDX query 1 is presented on the Fig. 11.

	Column Labels							
	Children Books		Computer Books		Cooking Books		Total Number	Total Sales Count
Row Labels	Number	Sales Count	Number	Sales Count	Number	Sales Count		
Olivia Gomez	6	4	37	15	3	2	46	21
England	6	4	37	15	3	2	46	21
Stamen Dimitrov	30	13	125	42	15	6	170	61
Bulgaria	30	13	125	42	15	6	170	61
Valeri Rodev	6	4	25	14	7	2	38	20
Turkey	6	4	25	14	7	2	38	20
Grand Total	42	21	187	71	25	10	254	102

FIG. 11. The drill-across operation using measures "Number" and "Sales Count"

The following MDX query 2 presents the opposite of the drill-across operation and the results of it - the first OLAP cube with measure "Number" and the dimensions "Books", "Bookshops" and "Location".

- *MDX query 2:*

SELECT NON EMPTY Hierarchize(DrilldownMember
({{{DrilldownLevel({[Books].[Hierarchy].[All]})}}},
{[Books].[Hierarchy].[Publisher].&[HarperCollins]}))
DIMENSION PROPERTIES PARENT_UNIQUE_NAME,
[Books].[Hierarchy].[Publisher].[Genre],
[Books].[Hierarchy].[Title].[Publisher] ON COLUMNS ,
NON EMPTY CrossJoin(Hierarchize(DrilldownMember
({{{DrilldownLevel({[Bookshops].[Hierarchy].[All]})}}},
{[Bookshops].[Hierarchy].[Regional Manager].&[Richard Gray]})),
Hierarchize({DrilldownLevel({[Location].[Hierarchy].[All]})}))
DIMENSION PROPERTIES PARENT_UNIQUE_NAME,
[Bookshops].[Hierarchy].[Regional Manager].[Owner],
[Bookshops].[Hierarchy].[Bookshop Name].[Regional Manager] ON ROWS
FROM [Bookshops3] WHERE ([Measures].[Number])
CELL PROPERTIES VALUE, FORMAT_STRING, LANGUAGE,
BACK_COLOR, FORE_COLOR, FONT_FLAGS

Result: The result of the MDX query 2 is presented on Fig. 12.

Number	Column Labels			
Row Labels	Children Books	Computer Books	Cooking Books	Grand Total
Olivia Gomez	**6**	**37**	**3**	**46**
England	6	37	3	46
Stamen Dimitrov	**30**	**125**	**15**	**170**
Bulgaria	30	125	15	170
Valeri Rodev	**6**	**25**	**7**	**38**
Turkey	6	25	7	38
Grand Total	**42**	**187**	**25**	**254**

FIG. 12. The first cube using measure "Number"

The following MDX query 3 presents the inverse of the drill-across operation and the results of it - the second OLAP cube with measure "Sales count" and the dimensions "Books", "Bookshops" and "Location".

- *MDX query 3:*

SELECT NON EMPTY Hierarchize(DrilldownMember
({{{DrilldownLevel({[Books].[Hierarchy].[All]})}}},
{[Books].[Hierarchy].[Publisher].&[HarperCollins]}))
DIMENSION PROPERTIES PARENT_UNIQUE_NAME,
[Books].[Hierarchy].[Publisher].[Genre],
[Books].[Hierarchy].[Title].[Publisher] ON COLUMNS,
NON EMPTY CrossJoin(Hierarchize(DrilldownMember
({{{DrilldownLevel({[Bookshops].[Hierarchy].[All]})}}},
{[Bookshops].[Hierarchy].[Regional Manager].&[Richard Gray]})),
Hierarchize({DrilldownLevel({[Location].[Hierarchy].[All]})}))

DIMENSION PROPERTIES PARENT_UNIQUE_NAME,
[Bookshops].[Hierarchy].[Regional Manager].[Owner],
[Bookshops].[Hierarchy].[Bookshop Name].[Regional Manager] ON ROWS
FROM [Bookshops3] WHERE ([Measures].[Sales Count])
CELL PROPERTIES VALUE, FORMAT_STRING, LANGUAGE,
BACK_COLOR, FORE_COLOR, FONT_FLAGS

Result: The result of the MDX query 3 is presented on the Fig. 13.

Sales Count	Column Labels			
Row Labels	Children Books	Computer Books	Cooking Books	Grand Total
Olivia Gomez	**4**	**15**	**2**	**21**
England	4	15	2	21
Stamen Dimitrov	**13**	**42**	**6**	**61**
Bulgaria	13	42	6	61
Valeri Rodev	**4**	**14**	**2**	**20**
Turkey	4	14	2	20
Grand Total	**21**	**71**	**10**	**102**

Fig. 13. The second cube using measure "Sales count"

4. Conclusion

In the presented paper it is proposed to expand the OLAP-cube data analysis by implementing some of the index matrix operations. Some of them operate with large aggregates of multidimensional data from two or more OLAP cubes. This paper is the fifth part of series of papers investigating the OLAP operations by index matrices. In the future the authors will finish the studies and some fields of application of the OLAP-analysis in multi-criteria system for effective performance management [15] will be discussed.

The scientific problem of OLAP-cube modeling, as well as the matrix-based interpretation of the analysis and retrieval of the information in it, has a wide practical application that makes it possible to work not only with precise parameters but also with fuzzy or intuitionistic fuzzy parameters.

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